Phys 404
Spring 2011
Homework 6, CHAPTER 4
Due Thursday, March 17, 2010 @ 12:30 PM

Chapter 4 assignment: Read chapter 4, then do these problems in chapter 4:

1. K+K, Chapter 4, Problem 1

2. K+K, Chapter 4, Problem 2

3. K+K, Chapter 4, Problem 5

4. K+K, Chapter 4, Problem 7

Problems in classical statistical mechanics:

5. A pendulum has length L and mass m, and makes an angle θ with the vertical direction. Assuming that the amplitude of oscillation is small, find $\langle\theta\rangle$, $\langle\theta^2\rangle$, $\langle v\rangle$, and $\langle v^2\rangle$ when the temperature of the pendulum is τ. **Hint**: Express the Hamiltonian in terms of the single coordinate (θ) and conjugate momentum (angular momentum ℓ) of the pendulum. Expand the potential for small oscillations, but take the limits of integration on θ in the partition function out to ±∞.

6. Referring to problem 5, what value of mL will give $(\langle\theta^2\rangle)^{1/2} = 0.001$ radian when the pendulum is at T = 300 K?

7. Consider a classical N-particle system with Hamiltonian given by

$$H = \sum_{i=1}^{N} (p_{xi}^2 + p_{yi}^2 + p_{zi}^2)/2m_i \quad + \quad V(\mathbf{r_1}, \mathbf{r_2}, \ldots \mathbf{r_N})$$

where V is the potential energy in terms of the coordinates of all the particles. Show that Z can be separated into kinetic and potential energy parts, $Z = Z_{kin}Z_{pot}$, and that the Z_{kin} term is the partition function for an ideal gas. This result is useful for studying the statistical mechanics of liquids, because Z_{pot} depends only on the positions of the particles.

General hints: Problems 5-7 involve classical statistical mechanics. In this case, the sums over quantum states are replaced by integrals over all position and momentum components, so that, for example,

$$\langle X\rangle = \frac{\frac{1}{h^{Nd}}\int\ldots\int X(\vec{p},\vec{q})e^{\left(\frac{-H(\vec{p},\vec{q})}{\tau}\right)}(d\vec{p})^N(d\vec{q})^N}{Z} \qquad Z = \frac{1}{h^{Nd}}\int\ldots\int e^{\left(\frac{-H(\vec{p},\vec{q})}{\tau}\right)}(d\vec{p})^N(d\vec{q})^N$$

Here N is the number of particles, and d is the spatial dimensionality, of the problem. Note that there is one integral for each component of momentum and position for each particle, so that there are 2Nd integral signs indicated by the $\int \ldots \int$ in the formulas.